FORCE SYSTEMS

CHAPTER OUTLINE

FORCE SYSTEMS

❖SECTION B THREE-DIMENSIONAL FORCE SYSTEMS

2/7 Rectangular Components

- Many problems in mechanics require analysis in three dimensions.
- And for such problems, it is often necessary to resolve a force into its three mutually perpendicular components.
- The force **F** acting at point *O* in Fig. 2/16 has the *rectangular components* F_x, F_y, F_z where. The unit vectors **i**, **j**, and **k** are in the *x*-, *y*-, and *z*-directions, respectively.

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$$F_x = F\cos\theta_x \qquad F = \sqrt{F_x{}^2 + F_y{}^2 + F_z{}^2}$$

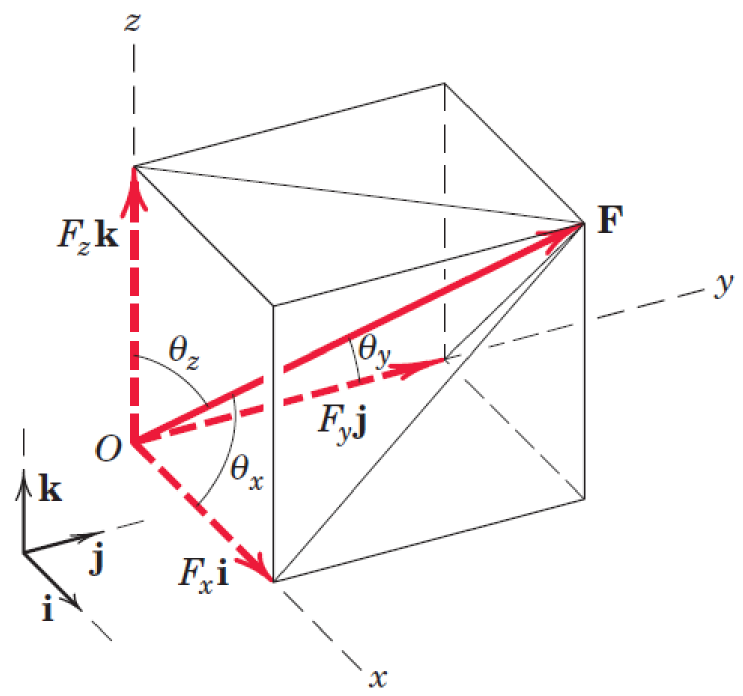
$$F_y = F\cos\theta_y \qquad \mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$F_z = F\cos\theta_z \qquad \mathbf{F} = F(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z)$$

The direction of a force is described by:-

(a) two points on the line of action of the force.

(*b*) two angles which orient the line of action.



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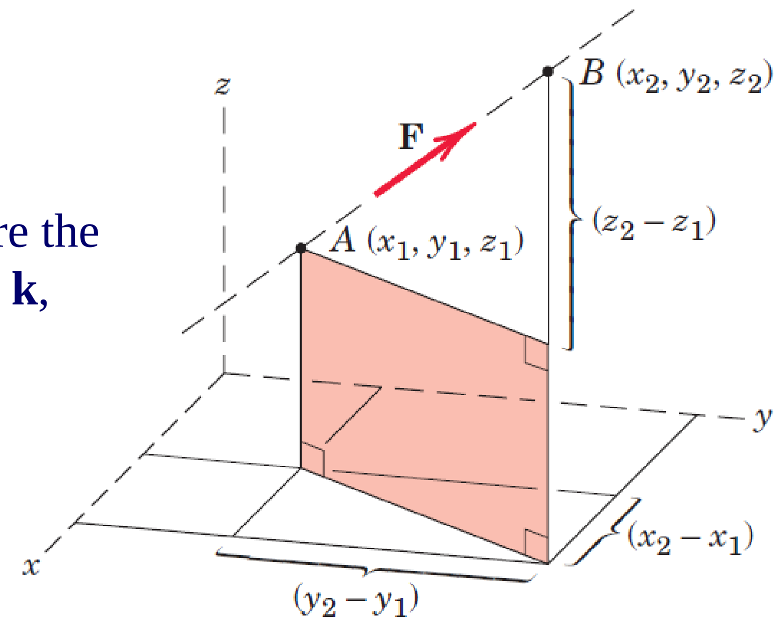
(a) two points on the line of action of the force.

If the coordinates of points *A* and *B* of Fig. 2/17 are known, the force **F** may be written as

$$\mathbf{F} = F\mathbf{n}_F = F\frac{\overrightarrow{AB}}{\overline{AB}} = F\frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

Thus the x, y, and z scalar components of **F** are the scalar coefficients of the unit vectors **i**, **j**, and **k**, respectively.



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(b) two angles which orient the line of action.

Consider the geometry of Fig. 2/18. We assume that the angles θ and ϕ are known. First resolve **F** into horizontal and vertical components.

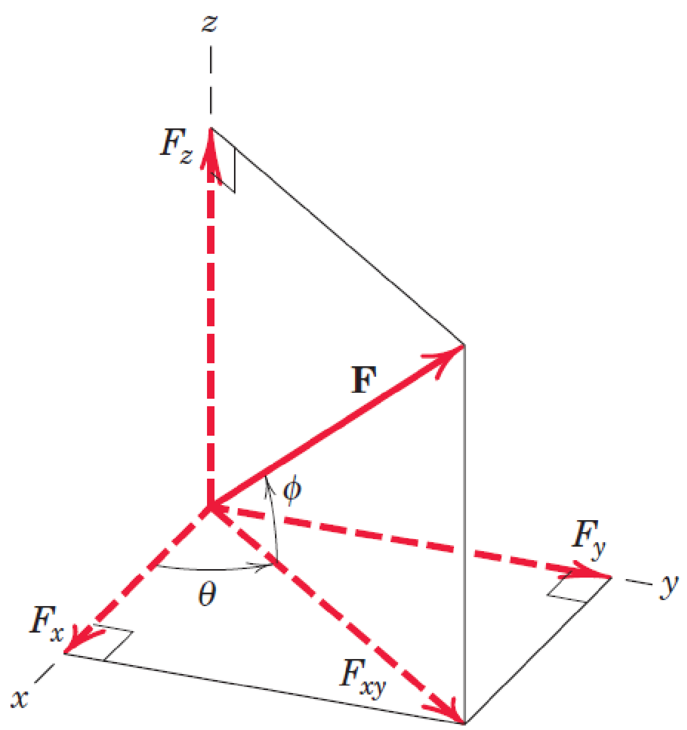
$$F_{xy} = F\cos\phi$$

$$F_z = F\sin\phi$$

Then resolve the horizontal component F_{xy} into x- and y-components.

$$F_x = F_{xy}\cos\theta = F\cos\phi\cos\theta$$

$$F_y = F_{xy}\sin\theta = F\cos\phi\sin\theta$$



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A force **F** with a magnitude of 100 N is applied at the origin *O* of the axes *x*-*y*-*z* as shown. The line of action of **F** passes through a point *A* whose coordinates are 3 m, 4 m, and 5 m. Determine (*a*) the *x*, *y*, and *z* scalar components of **F**, (*b*) the projection F_{xy} of **F** on the *x*-*y* plane, and (*c*) the projection F_{OB} of **F** along the line *OB*.

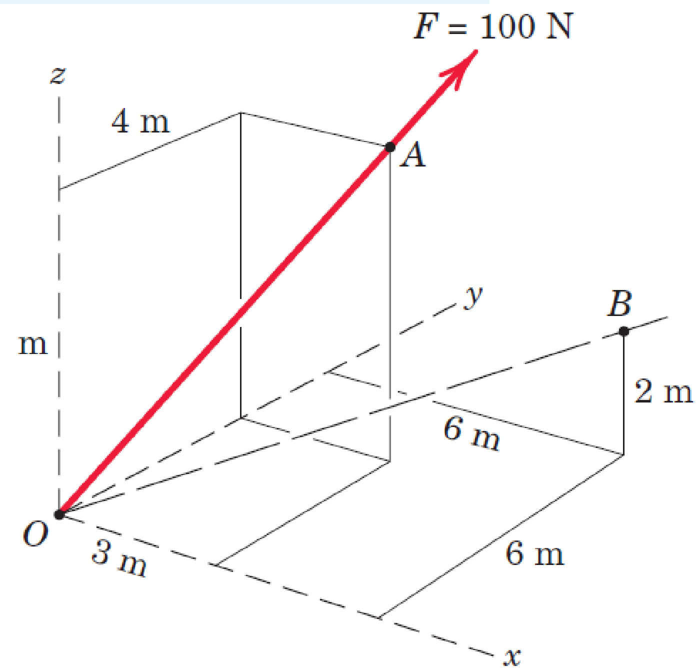
Solution. ***Part (a).*** We begin by writing the force vector **F** as its magnitude *F* times a unit vector $\mathbf{n}_{OA}$.

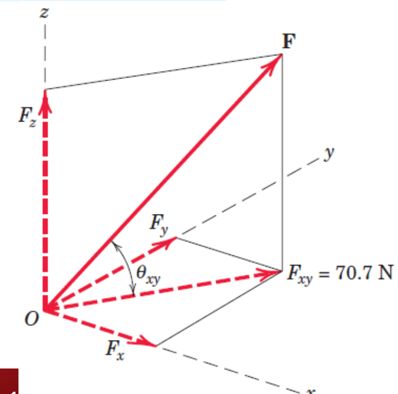
$$\mathbf{F} = F\mathbf{n}_{OA} = F\frac{\overrightarrow{OA}}{\overline{OA}} = 100\left[\frac{3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}}{\sqrt{3^2 + 4^2 + 5^2}}\right]$$

$$= 100[0.424\mathbf{i} + 0.566\mathbf{j} + 0.707\mathbf{k}]$$

$$= 42.4\mathbf{i} + 56.6\mathbf{j} + 70.7\mathbf{k}\ \text{N}$$

$F_x = 42.4$ N $\qquad F_y = 56.6$ N $\qquad F_z = 70.7$ N

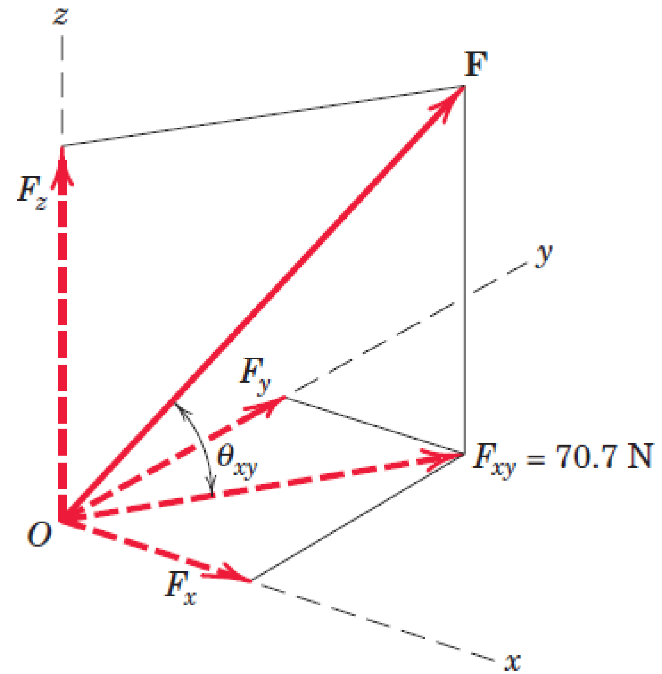




Part (b). The cosine of the angle θ_{xy} between **F** and the x-y plane is

$$\cos\theta_{xy} = \frac{\sqrt{3^2+4^2}}{\sqrt{3^2+4^2+5^2}} = 0.707$$

so that $F_{xy} = F\cos\theta_{xy} = 100(0.707) = 70.7\text{ N}$



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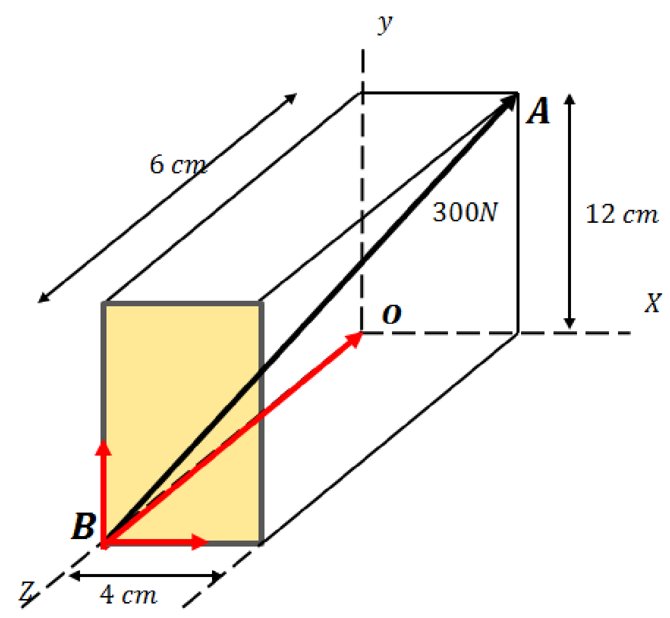
Ex:- Determine the *x*, *y*, and *z* vectors components of F, and scalar components.

Solution

O (0, 0, 0), B(0, 0,6), A(4, 12, 0)



$$F_{BA} = \overrightarrow{F_{BA}} \; . \, n_{BA}$$

$$F_{BA} = 300 * \left(\frac{(4-0)i+(12-0)j+(0-6)k}{\sqrt{(4-0)^2+(12-0)^2+(0-6)^2}}\right)$$

$$F_{BA} = 300 * \left(\frac{4i+12j-6k}{\sqrt{4^2+12^2+6^2}}\right)$$

$$F_{BA} = 300 * \left(\frac{4i+12j-6k}{14}\right)$$

F_{BA}= 300* (0.285i + 0.857j - 0.428k)

F_{BA}= 85.7i + 257.1j – 128.4k

scalar components of F =

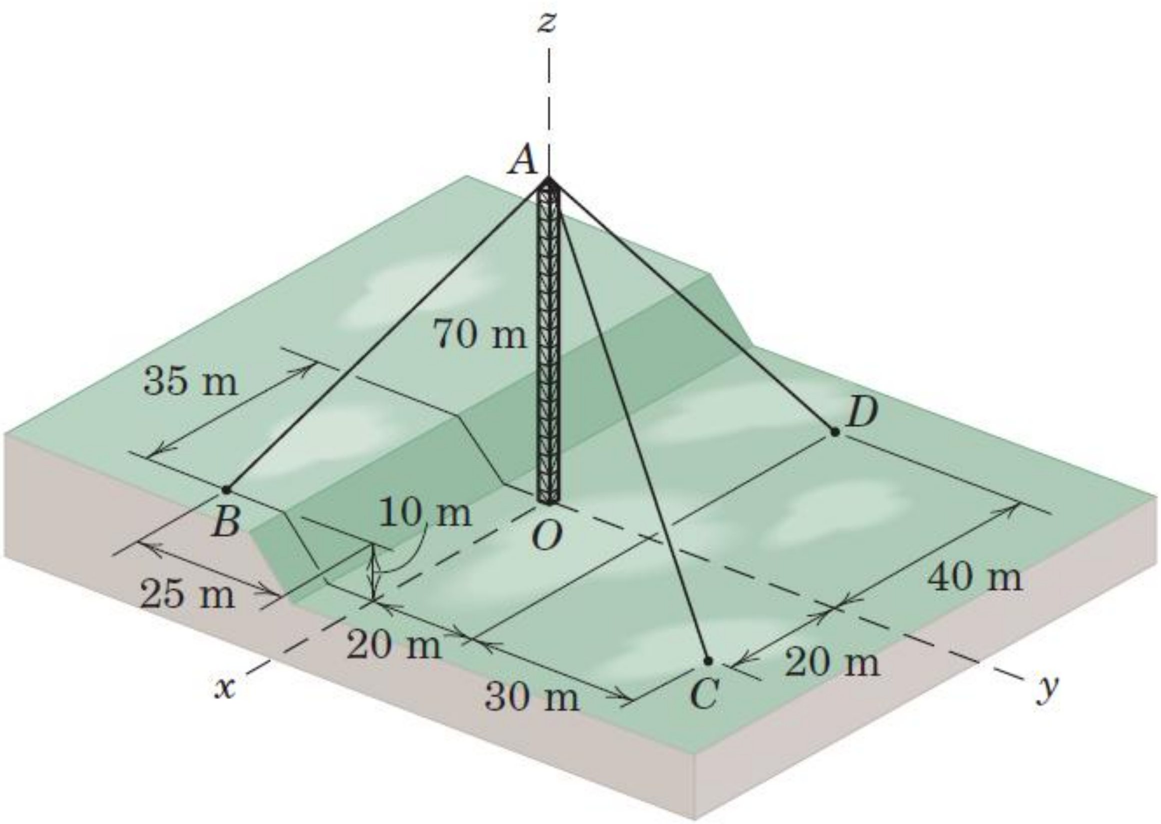
F_x=85.5 N

F_y= 257.1N

$F_z = 128.4N$

H.w

The ***70-m*** microwave transmission tower is steadied by three guy cables as shown. Cable ***AB*** carries a tension of ***12 kN***. Express the corresponding force on point ***B*** as a vector.



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Q2/ Express the force **F** as a vector in terms of the unit vectors **i, j, and k.** Determine the angles ***x, y, and z*** which **F** makes with the positive *x*-, *y*-, and *z*-axes.